

Hope you had a great Halloween! ☺

Group 1:

$$\#1. \int 9x^4 + 11x^{-3} - \frac{1}{2} \cdot \frac{1}{x} + 4x^{\frac{1}{2}} - 8 dx$$

$$= 9 \frac{x^5}{5} + 11 \frac{x^{-2}}{-2} - \frac{1}{2} \ln|x| + 4 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 8x + C$$

$$\#2. \int (x+1)^2 dx = \int x^2 + 2x + 1 dx = \frac{x^3}{3} + 2 \frac{x^2}{2} + x + C$$

$$\#3. \int \frac{x^3 + 3x^6 + 1}{x^4} dx = \int \frac{1}{x} + 3x^2 + x^{-4} dx = \ln|x| + x^3 + \frac{x^{-3}}{-3} + C$$

$$\#4. \int_3^1 f(x) dx + \int_1^4 f(x) dx + \int_3^4 f(x) dx + \int_4^5 2f(x) dx$$

Denote the antiderivative of  $f(x)$  as  $F(x)$ . Then by Fundamental thm of calculus  
(or also called "net change thm")

$$? = \cancel{F(1)} - \cancel{F(3)} + \cancel{F(4)} - \cancel{F(1)} + \cancel{F(4)} - \cancel{F(3)} + 2F(5) - 2\cancel{F(4)}$$

$$= 2F(5) - 2F(3) = \int_3^5 2f(x) dx$$

Group 2:

$$1. \int (2x-3)^{15} dx$$

$$u = 2x-3$$

$$du = 2dx \Rightarrow dx = \frac{du}{2}$$

$$= \int u^{15} \frac{du}{2}$$

$$= \frac{1}{2} \int u^{15} du = \frac{1}{2} \frac{u^{16}}{16} + C = \frac{1}{2} \frac{(2x-3)^{16}}{16} + C$$

$$2. \int 3xe^{x^2} dx$$

$$u = x^2$$

$$du = 2x dx \Rightarrow \frac{du}{2} = x dx$$

$$= \int 3e^{x^2} \underbrace{x dx}_{\frac{du}{2}}$$

$$= \int 3e^u \frac{du}{2}$$

$$= \frac{3}{2} \int e^u du = \frac{3}{2} e^u + C = \frac{3}{2} e^{x^2} + C$$

$$\begin{aligned} \#3 \quad \int \frac{x^4}{x^5+3} dx & \quad u = x^5+3 \\ & \quad du = 5x^4 dx \Rightarrow \frac{du}{5} = x^4 dx \\ & = \int \frac{1}{u} \frac{du}{5} \\ & = \frac{1}{5} \ln|u| + C = \frac{1}{5} \ln|x^5+3| + C \end{aligned}$$

$$\begin{aligned} \#4. \quad \int x(x^2-1)^3 dx & \quad u = x^2-1 \\ & \quad du = 2x dx \Rightarrow \frac{du}{2} = x dx \\ & = \int u^3 \frac{du}{2} \\ & = \frac{1}{2} \frac{u^4}{4} + C = \frac{1}{2} \frac{(x^2-1)^4}{4} + C \end{aligned}$$

$$\begin{aligned} \#5. \quad \int x^4(3-5x^5)^{\frac{1}{3}} dx & \quad u = 3-5x^5 \\ & \quad du = -25x^4 dx \Rightarrow -\frac{du}{25} = x^4 dx \\ & = \int u^{\frac{1}{3}} \frac{du}{-25} \\ & = -\frac{1}{25} \int u^{\frac{1}{3}} du = -\frac{1}{25} \frac{u^{\frac{4}{3}}}{\frac{4}{3}} + C = -\frac{1}{25} \frac{3}{4} (3-5x^5)^{\frac{4}{3}} + C \end{aligned}$$

$$\begin{aligned} \#6. \quad \int \frac{2-x}{\sqrt{2x^2-8x+1}} dx & \quad u = 2x^2-8x+1 \\ & \quad du = (4x-8) dx = 4(x-2) dx \Rightarrow \frac{du}{4(x-2)} = dx \\ & = \int \frac{-(x-2)}{\sqrt{u}} \frac{du}{4(x-2)} \\ & = -\frac{1}{4} \int \frac{du}{\sqrt{u}} = -\frac{1}{4} u^{-\frac{1}{2}} du = -\frac{1}{4} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C = -\frac{1}{2} u^{\frac{1}{2}} + C \\ & = -\frac{1}{2} (2x^2-8x+1)^{\frac{1}{2}} + C \end{aligned}$$